

УДК 519.6

**ALGORITHMS USED IN TIM CODE TO CONTROL VELOCITY OF  
DETONATION FRONT PROPAGATION**S. S. Sokolov, A. A. Pushkaryev, V. N. Motlokhov  
(FSUE "RFNC-VNIIEF", Sarov N.Novgorod region)

The paper describes three algorithms used to control the velocity of the HE detonation front propagation, which were developed for unstructured polygonal and polyhedral grids. The first of them is the *exact* control algorithm, when the detonation time for all HE-containing cells of the grid is determined once at the beginning of computations. The second is the *step-by-step* control algorithm that allows specifying in the computation process the time of the detonation wave arrival at each cell using the times of its arrival at neighboring cells. Both of the algorithms are cost-effective; however, they have certain restrictions for a wide range of applied problems. The third algorithm represents by itself an improved version of the step-by-step control algorithm. In this algorithm the accuracy of calculating the detonation time for each HE-containing cell is improved, because the direction of the moving detonation wave front is taken into account. In contrast to the basic step-by-step control algorithm, where the cell detonation time is corrected in the process of successively considering each detonated neighboring cell, in the third algorithm the detonation time for a given cell is corrected by considering neighbor cells from the first layer of cells surrounding the given cell. The third one is the general-purpose algorithm that can be applied for calculations with the HE detonation control in regions of complex geometries; however, this is a time consuming algorithm in comparison with the first two algorithms. To demonstrate the applicability of all algorithms, the paper presents numerical results for several methodological problems on the simulation of a detonation wave propagating in HE using the TIM and TIM-2D codes for solving continuum mechanics problems on unstructured polygonal and polyhedral grids with an arbitrary number of links at nodes.

*Ключевые слова:* TIM code, explosive, steady detonation, detonation front, detonation velocity, unstructured grids.

**Introduction**

TIM-2D and TIM [1, 2] are codes intended for Lagrangian multidimensional continuum mechanics simulations on polygonal and polyhedral unstructured grids. They enable simulations on arbitrary-connectivity grids (with an arbitrary number of adjacent cells and edges).

The propagation of a steady detonation wave in a high explosive (HE) in TIM-2D and TIM is simulated using the Chapman—Jouguet model with controlled detonation front velocity [3]. The control algorithms are divided into two types: *exact* (using different kinds of involutes) and *step-by-step* control algorithms [4]. The algorithms of both types calculate the times of detonation front arrival at HE cell centers.

The philosophy of the exact control is as follows. Based on the initial surfaces or centers of initiation and problem geometry, the algorithms identify the shortest DF paths and respective distances from the initiation centers to the HE cells. Following this, considering the velocity of detonation, the algorithms find the times of DF arrival at the cell centers.

The step-by-step control algorithm differs from the exact control algorithm in that the time of the steady detonation arrival at the corresponding grid point is calculated and can be corrected in the process of computation. Knowing the initial surfaces or centers of initiation and the cells detonated at the previous

time step, one can determine the times of DF arrival at the centers of the cells’ HE-containing neighbors at the next time step. This algorithm employs an approach, in which the times of DF arrival at a given cell are determined based on the time of DF arrival at its neighbors. Some step-by-step control algorithms for the case of structured grids have been discussed earlier in Refs. [5–7].

### Applications, strengths and drawbacks of control algorithms

Which of the algorithms to use depends on the HE region geometry and presence of an inert layer conveying the detonation. For example, Fig. 1 shows a geometry, for which one can use the point-wise control algorithms. Here, the HE region is bounded above, on the right and left by the straight lines, and below, by the sine curve. The center of initiation is located in the left bottom corner.

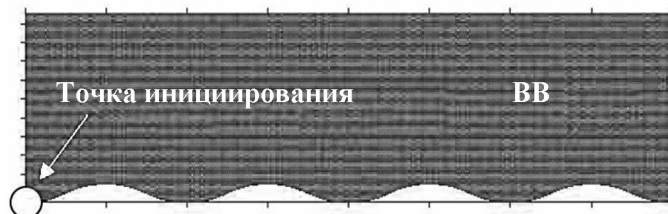


Fig. 1. Example of geometry, for which one can use the exact control algorithms

Fig. 2, on the opposite, shows a geometry, for which the exact control algorithms are inappropriate. Here, HE is present in two regions separated by an inert layer. Initiation is set along the  $AB$ ,  $CD$  and  $EF$  segments of the outer boundary. One can see that for the upper HE layer one can use exact control, while for the lower layer it is unsuitable, because detonation caused by the shock action translated from the upper HE layer through the inert layer may occur sooner than detonation from the given surface of initiation in this region.

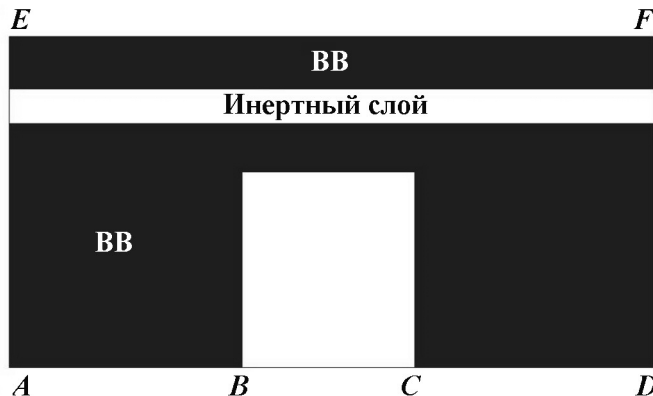


Fig. 2. Example of geometry, for which one cannot use the exact control algorithms

The drawbacks of the exact control algorithm include the complexity of boundary description for the case of a large number of *dark zones* (HE zones beyond the visual line of sight from the centers of initiation) in the initial geometry, assumption of constant detonation velocity, and inapplicability of the algorithm in the presence of an inert layer conveying the detonation. The advantages, on the other side, include exact calculations of detonation times for relatively simple geometries of bodies containing HE.

The advantages of the step-by-step control algorithm include its independence of the complexity of the initial HE region geometry, absence of the assumption of constant detonation velocity in different HE layers, and possibility of detonation transfer through the inert layer. Among its drawbacks is the non-zero error in detonation time calculations.

### Exact control for two-dimensional simulations

In two-dimensional simulations involving the exact control algorithm, the detonation time calculation procedure is performed once at the beginning of a computation and the times of detonation front arrival are calculated for each cell.

The general sequence of steps of the detonation time calculation algorithm for all the cells used in the exact detonation front control is the following:

1. Specify a set of initiation points (the initiation surface is approximated by a discrete set of points).
2. If the system contains dark HE zones invisible from the points of initiation, create an envelope for the HE region defined by its boundary nodes.

3. For each HE cell, calculate the minimum distance to the set of initiation points rounding the curved boundaries (outer or specifying the inert materials).
4. Use the detonation velocity and the calculated distance to identify the time of detonation arrival for each cell.

The algorithm for calculating the distance from the current point belonging to the set of initiation points to the HE cell center with the rounding of the curved HE boundaries consists of the following steps:

1. Identify whether the cell is visible from the initiation point.
2. If not, then
  - a) find the tangent (points of contact 1) to the envelope from the initiation point or the nearest envelope deflection point (based on a change of sign of the radius vector derivative from the initiation point to the curve point) and the distance from the initiation point to the point of contact 1;
  - b) find the tangent (points of contact 2) to the envelope from the cell center or the envelope deflection point nearest to the cell center and the distance from the cell center to the point of contact 2;
  - c) find the length of the envelope boundary segment between points of contact 1 and 2;
  - d) find the distance traveled by the detonation from the cell center as a sum of the distances found at substeps.  $a-d$ .
3. If the cell center is visible, i.e. the HE boundary does not cross the segment of the straight line between the initiation center and the cell center, then the length of this segment is the sought distance.

Next, at each step of a computation (for all HE cells), the detonation time calculated by the exact control algorithm is compared with the detonation times already calculated for the other points in the set of the initiation points, and the minimum time is picked out.

### Step-by-step control for two-dimensional and three-dimensional simulations

In applied simulations, where the envelope for the HE region cannot be described or where detonation-conveying inert layers are present, it becomes hardly possible to use the exact control algorithms on unstructured polygonal and polyhedral grids. In this case, TIM-2D and TIM employ the step-by-step control algorithms. Let us consider the step-by-step control algorithm in more detail.

Let us call all the cells having one common edge with the cell under consideration the cell's first neighbor layer. An example of the first layer is shown in Fig. 3, *a* (for simplicity, we consider a two-dimensional

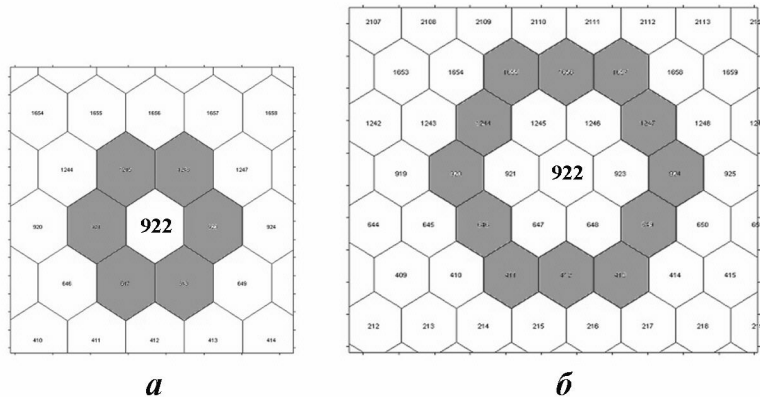


Рис. 3. Примеры первой (*a*) и второй (*б*) клеточных соседних слоев 922

case). The dark cells constitute the first neighbor layer of cell 922 (cell 922 itself does not belong to the neighborhood).

Let us define the second neighbor layer in the following way. We take all the cells of the first neighbor layer of the cell under consideration and the first neighbor layer for each cells of these cells. The set of all the cells involved, except from the starting cell and its first neighbor layer, is called the cell’s second neighbor layer. An example of the second layer is shown in Fig. 3, *b*. The dark cells constitute the second neighbor layer for cell 922. The cell’s third neighbor layer (required for three-dimensional simulations) is defined similarly to the second layer.

The step-by-step control algorithm (for the time variable) includes the following steps:

1. Cells from a list known in advance (from the problem definition) are declared detonated, and their detonation times are assumed equal to the times of initiation of corresponding initiation points (in accordance with the initial conditions). For example, if we set instantaneous initiation on the outer surface of a domain, all the cells adjacent to this boundary are declared detonated at the initial time, and this time is considered to be their detonation time.
2. At each time step in a computation, detonation front velocity control is performed for each not yet detonated cell. The cell’s neighborhood is also considered (the first layer, or the first and the second one, or the second and the third one). Respective detonation times are considered for all the neighbor cells, and the already-detonated cells or the cells to detonate at the current time step are picked out. Among the picked cells, the algorithm identifies the cells, the detonation from which managed to arrive at the current cell over the time under consideration. The shortest time of detonation arrival from such cells at the current cell is taken as the cell’s detonation time.

**Correction of step-by-step control  
by considering the direction of motion of the detonation front**

Note that the accuracy of capturing the propagation of the detonation front by the step-by-step algorithm on unstructured polygonal or polyhedral grids is relatively low. Therefore, in order to improve the accuracy of the detonation time calculations involving the step-by-step control method, an additional algorithm has been developed. The philosophy of the algorithm is the following.

If the detonation time of the current cell for the step-by-step control is calculated as the time of detonation arrival from one of the cell’s neighbors, then the control correction is performed by considering the direction of motion of the detonation front, and the cell’s detonation time is corrected by considering its neighbor cells in the first neighbor layer in pairs [4].

**Detonation time correction algorithm for the two-dimensional case.** Suppose we need to find the time of detonation front arrival at a cell having its center at the point  $C$  (cell  $C$ ) using the known times of detonation front arrival at cells having their centers at the points  $C_1$  and  $C_2$  (cells  $C_1$  and  $C_2$ ) (Fig. 4).

The idea of such correction is the following. Using the known times  $t_{C_1}$  and  $t_{C_2}$  of detonation arrival at the cells  $C_1$  and  $C_2$  and assuming that the time variation of detonation motion from  $C_1$  to  $C_2$  is linear, we can identify the time of detonation front arrival at any point  $K$  of the segment  $C_1C_2$  by linear interpolation. Taking the distance  $|C_1K|$  as parameter  $r$  (see. Fig. 4) and using this parameter to find the minimum of the function  $t_K + |CK|/D$ , where  $D$  — is the

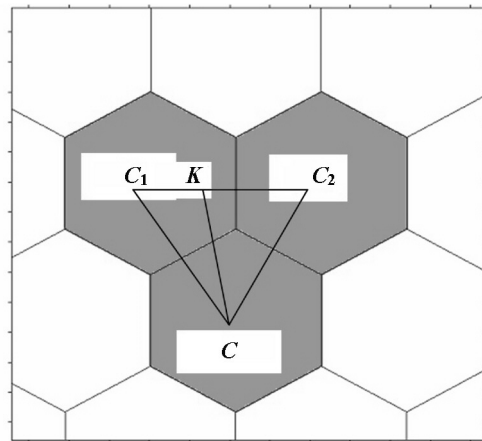


Fig. 4. Control correction by considering the direction of motion of the detonation front in the two-dimensional case

value of the detonation velocity, we correct the time of detonation front arrival at cell  $C$  from the pair of cells  $C_1$  and  $C_2$ . Taking the first neighbor layer for the given cell and performing the correction for each pair of cells in this layer, we find the minimum detonation time for this cell.

Let us describe the algorithm to find the time of detonation from a pair of cells in more detail:

1. Denoting  $R = |C_1C_2|$ ,  $r = |C_1K|$ , we find the distance  $|CK|$  using the law of cosines for the triangle  $CC_1K$ :

$$|CK| = \sqrt{r^2 - ar + \beta}, \quad \text{where} \quad a = 2 \frac{\overrightarrow{C_1C_2}}{|R|} \cdot \overrightarrow{C_1C}, \quad \beta = |CC_1|^2.$$

2. Let us represent the detonation time at the point  $K$  as some function  $f_1(r) = \gamma r + t_{C_1}$ , where  $\gamma = (t_{C_2} - t_{C_1})/R$ .
3. Consider the function  $f(r) = f_1(r) + |CK|/D = \gamma r + t_{C_1} + \sqrt{r^2 - ar + \beta}/D$ . Let us examine it for the minimum:

$$f(r) \xrightarrow{r \in [0, R]} \min \Rightarrow \frac{df(r)}{dr} = \frac{2r - a}{2D\sqrt{r^2 - ar + \beta}} + \gamma = 0 \Rightarrow 2\gamma D\sqrt{r^2 - ar + \beta} = a - 2r.$$

The equation can have a solution only at  $\gamma(a - 2r) \geq 0$ . Taking its square, we obtain

$$Ar^2 + Br + E = 0, \quad \text{where} \quad A = 4\gamma^2 D^2 - 4; \quad B = 4a - 4\gamma^2 D^2 a; \quad E = 4\gamma^2 D^2 \beta - a^2.$$

Depending on the sign of the discriminant, the equation has one root, two roots or no root. If the equation has roots, then only those are picked out, which satisfy the conditions  $\gamma(a - 2r) \geq 0$  and  $r \in [0, R]$ .

4. We choose the least of the values  $f(0)$ ,  $f(R)$ ,  $f(r_1)$ ,  $f(r_2)$ , where  $r_1$  and  $r_2$  are suitable equation roots, if they exist.

The result is taken as the detonation time for the current cell having its center at the point  $C$ .

**Detonation time correction algorithm for the three-dimensional case.** Suppose we need to find the time of detonation front arrival at a cell having its center at the point  $C$  using the known times of detonation front arrival at cells having their centers at  $C_1$ ,  $C_2$  and  $C_3$  (Fig. 5).

Using the known times  $t_{C_1}$ ,  $t_{C_2}$  and  $t_{C_3}$  and assuming that the time variation of detonation motion in the triangle  $C_1C_2C_3$  is linear, we can identify the time of detonation front arrival at any point  $K$  in the plane by interpolation. Taking the weight factors of the radius vectors  $\vec{r}_{C_1}$ ,  $\vec{r}_{C_2}$  and  $\vec{r}_{C_3}$  as parameters  $\alpha$ ,  $\beta$ ,  $\gamma$  of the points  $C_1$ ,  $C_2$  and  $C_3$  for the point  $K$  ( $\vec{r}_K = \alpha\vec{r}_{C_1} + \beta\vec{r}_{C_2} + \gamma\vec{r}_{C_3}$ ) and considering that the time of detonation front arrival at the point  $K$  is calculated by the formula  $t_K = \alpha t_{C_1} + \beta t_{C_2} + \gamma t_{C_3}$ , we find the minimum of the function  $t_K + |CK|/D$  ( $D$  – is the value of detonation velocity) using these parameters and correct the time of detonation front arrival at the cell  $C$  from three cells  $C_1$ ,  $C_2$  and  $C_3$ . Taking the first neighbor layer for the given cell and performing the correction for each triple of cells in this layer, we find the minimum detonation time for this cell.

Let us describe the formulas to find the detonation time from a triple of cells in more detail:

1. For an arbitrary point  $K$  lying in the plane  $C_1C_2C_3$  inside the triangle  $C_1C_2C_3$ , it holds for the introduced notation that  $\vec{r}_K = \alpha\vec{r}_{C_1} + \beta\vec{r}_{C_2} + \gamma\vec{r}_{C_3}$ , where  $\alpha + \beta + \gamma = 1$  и  $\alpha \geq 0$ ,  $\beta \geq 0$ ,  $\gamma \geq 0$ . Hence,  $\vec{r}_K = \alpha(\vec{r}_{C_1} - \vec{r}_{C_3}) + \beta(\vec{r}_{C_2} - \vec{r}_{C_3}) + \vec{r}_{C_3}$ .

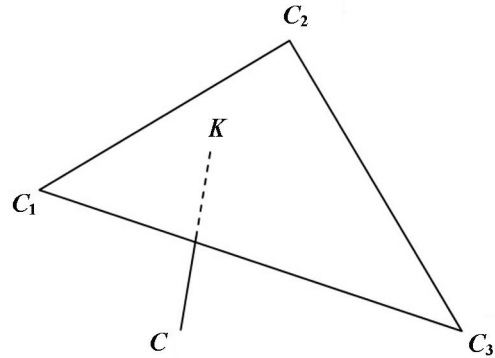


Fig. 5. Control correction by considering the direction of motion of the detonation front in the three-dimensional case

2. Calculate the time of detonation arrival at the point  $K$  by the following formula:

$$t_K = \alpha t_{C_1} + \beta t_{C_2} + \gamma t_{C_3} = \alpha t_{C_1} + \beta t_{C_2} + (1 - \alpha - \beta)t_{C_3} = \alpha(t_{C_1} - t_{C_3}) + \beta(t_{C_2} - t_{C_3}) + t_{C_3}.$$

3. Denote  $\vec{r}_{C_1} = (x_1, y_1, z_1)$ ,  $\vec{r}_{C_2} = (x_2, y_2, z_2)$ ,  $\vec{r}_{C_3} = (x_3, y_3, z_3)$ ,  $\vec{r}_C = (x_0, y_0, z_0)$ ,  $\vec{r}_K = (x_K, y_K, z_K)$ . Then,

$$\vec{r}_K = (x_K, y_K, z_K) = \left( \alpha(x_1 - x_3) + \beta(x_2 - x_3) + x_3, \alpha(y_1 - y_3) + \beta(y_2 - y_3) + y_3, \alpha(z_1 - z_3) + \beta(z_2 - z_3) + z_3 \right).$$

4. Represent the detonation time at the point  $C$  as some function  $f(\alpha, \beta) = t_K + |CK|/D$ .

5. Find

$$|CK|^2 = [\alpha(x_1 - x_3) + \beta(x_2 - x_3) + x_3 - x_0]^2 + [\alpha(y_1 - y_3) + \beta(y_2 - y_3) + y_3 - y_0]^2 + [\alpha(z_1 - z_3) + \beta(z_2 - z_3) + z_3 - z_0]^2 = A\alpha^2 + B\beta^2 + 2M\alpha\beta + 2N\alpha + 2P\beta + L,$$

where

$$\begin{aligned} A &= |C_1C_3|^2; & B &= |C_2C_3|^2; & L &= |CC_3|^2; \\ M &= (x_1 - x_3)(x_2 - x_3) + (y_1 - y_3)(y_2 - y_3) + (z_1 - z_3)(z_2 - z_3); \\ N &= (x_1 - x_3)(x_3 - x_0) + (y_1 - y_3)(y_3 - y_0) + (z_1 - z_3)(z_3 - z_0); \\ P &= (x_2 - x_3)(x_3 - x_0) + (y_2 - y_3)(y_3 - y_0) + (z_2 - z_3)(z_3 - z_0). \end{aligned}$$

6. Examine the function

$$f(\alpha, \beta) = t_K + \frac{|CK|}{D} = \alpha(t_1 - t_3) + \beta(t_2 - t_3) + t_3 + \frac{\sqrt{A\alpha^2 + B\beta^2 + 2M\alpha\beta + 2N\alpha + 2P\beta + L}}{D}$$

for the minimum ( $f(\alpha, \beta) \rightarrow \min$ ) given that  $\alpha \geq 0$ ,  $\beta \geq 0$ ,  $\alpha + \beta \leq 1$ .

Consider the function

$$\begin{aligned} g(\alpha, \beta) &= D(f(\alpha, \beta) - t_3) = D\alpha(t_1 - t_3) + D\beta(t_2 - t_3) + \sqrt{A\alpha^2 + B\beta^2 + 2M\alpha\beta + 2N\alpha + 2P\beta + L} = \\ &= \alpha T_1 + \beta T_2 + \sqrt{A\alpha^2 + B\beta^2 + 2M\alpha\beta + 2N\alpha + 2P\beta + L}, \end{aligned}$$

where  $T_1 = D(t_1 - t_3)$ ;  $T_2 = D(t_2 - t_3)$ .

It is evident that the minimum points  $g(\alpha, \beta)$  are the minimum points  $f(\alpha, \beta)$ . Find the minimum points  $g(\alpha, \beta)$ . Draw up the Lagrange function  $L(\alpha, \beta) = \lambda_1 g(\alpha, \beta) + \lambda_2(\alpha + \beta - 1) + \lambda_3(-\alpha) + \lambda_4(-\beta)$ , where not all of  $\lambda_i$  are zero at once. Necessary conditions for its minimum:

– steadiness

$$L_\alpha(\alpha, \beta) = 0; \quad L_\beta(\alpha, \beta) = 0,$$

i. e.

$$\lambda_1 g_\alpha(\alpha, \beta) + \lambda_2 - \lambda_3 = 0; \quad \lambda_1 g_\beta(\alpha, \beta) + \lambda_2 - \lambda_4 = 0; \quad (1)$$

– complementary slackness

$$\lambda_2(\alpha + \beta - 1) = 0; \quad \lambda_3(-\alpha) = 0; \quad \lambda_4(-\beta) = 0; \quad (2)$$

– non-negativity

$$\lambda_i \geq 0, \quad i = 1, \dots, 4. \quad (3)$$

Without details of calculations, let us write the complete result of solving the system (1)–(3) composed of seven parts:

- 1)  $\alpha = 0, \beta = 0$ , if  $g_\alpha(0, 0) \geq 0$  и  $g_\beta(0, 0) \geq 0$ ;
- 2)  $\alpha = \alpha_1, \beta = 0$ , where  $\alpha_1$  is found based on the condition  $g_\alpha(\alpha_1, 0) = 0, g_\beta(\alpha_1, 0) \geq 0$ ;
- 3)  $\alpha = 1, \beta = 0$ , if  $g_\alpha(1, 0) \leq 0, -g_\alpha(1, 0) + g_\beta(1, 0) \geq 0$ ;
- 4)  $\alpha = \alpha_1, \beta = \beta_1$ , where  $\alpha_1, \beta_1$  satisfy the system 
$$\begin{cases} g_\alpha(\alpha_1, \beta_1) = 0; \\ g_\beta(\alpha_1, \beta_1) = 0; \end{cases}$$
- 5)  $\alpha = \alpha_1, \beta = \beta_1$ , where  $\alpha_1, \beta_1$  satisfy the system 
$$\begin{cases} \alpha_1 + \beta_1 = 1; \\ g_\alpha(\alpha_1, \beta_1) = g_\beta(\alpha_1, \beta_1); \end{cases}$$
- 6)  $\alpha = 0, \beta = 1$ , if  $g_\beta(0, 1) \leq 0, g_\alpha(0, 1) - g_\beta(0, 1) \geq 0$ ;
- 7)  $\alpha = 0, \beta = \beta_1$ , where  $\beta_1$  is found from the conditions  $g_\beta(0, \beta_1) = 0, g_\alpha(0, \beta_1) \geq 0$ .

Based on these solutions, we identify the minimum of the function  $g(\alpha, \beta)$  and the minimum of the function  $f(\alpha, \beta) = g(\alpha, \beta) / D + t_3$ , too. The resulting value is taken as the detonation time for the cell having its center at the point  $C$ .

### Results of methodological computations

Consider a test problem of a propagating detonation wave depending on the way of its initiation for the two- and three-dimensional cases. The test problem geometry for the two- and three-dimensional cases in the section plane  $Oxy$  is represented as a  $5\text{ cm} \times 6\text{ cm}$  rectangle, as shown in Fig. 6. The region filled with an inert material is shown as a  $2\text{ cm} \times 1\text{ cm}$  rectangle located at a height of  $2\text{ cm}$  over the  $Ox$  axis; the remaining part is filled with an explosive. The detonation front velocity is  $D = 8,83\text{ km/s}$ . In the three-dimensional case, the test system has a thickness of  $1\text{ cm}$ . To simulate the HE region we used a grid of 2900 polygons in the two-dimensional case, and of 28300 polygons in the three-dimensional case.

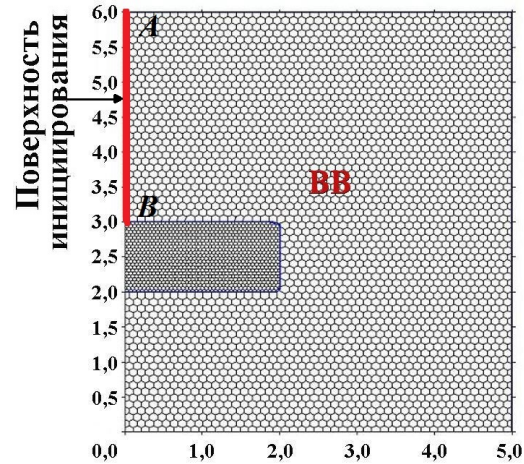


Fig. 6. Benchmark geometry

**Propagation of a plane-front detonation wave considering its passing over the inert layer.** Initiation is set along the segment  $AB$  of the left upper boundary (see. Fig. 6). Fig. 7 shows detonation front positions at the characteristic times calculated by the exact and step-by-step control algorithms. The solution of this problem is presented in the form

$$T(x_0, y_0) = \begin{cases} \frac{x_0}{D} & \text{for } x_0 \in [0, 5], y_0 \in [3, 6]; \\ \frac{2 + \sqrt{(x_0 - 2)^2 + (y_0 - 3)^2}}{D} & \text{for } x_0 \in [2, 5], y_0 \in [0, 3]; \\ \frac{3 + \sqrt{(x_0 - 2)^2 + (y_0 - 2)^2}}{D} & \text{for } x_0 \in [0, 2], y_0 \in [0, 2]. \end{cases}$$

Fig. 8 shows contour graphs of the detonation times calculated by the control algorithms and analytically. The relative error of the detonation times calculated by the control algorithms does not exceed 0,5% for all grid cells.

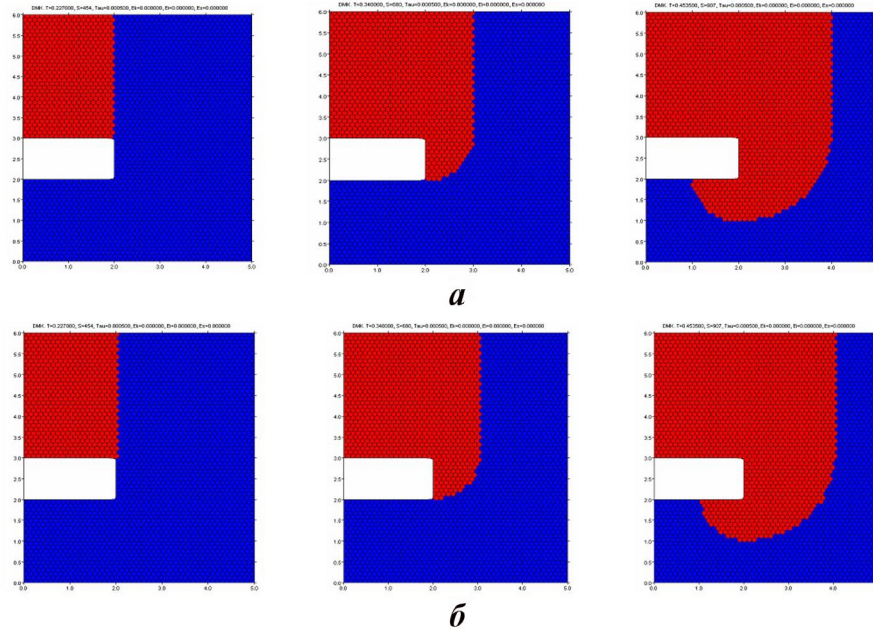


Рис. 7. Detonation front positions at characteristic times calculated by the exact- (a) and step-by-step (b) control algorithms: left —  $t = 0,227$ ; in the middle —  $t = 0,34$ ; right —  $t = 0,4535$

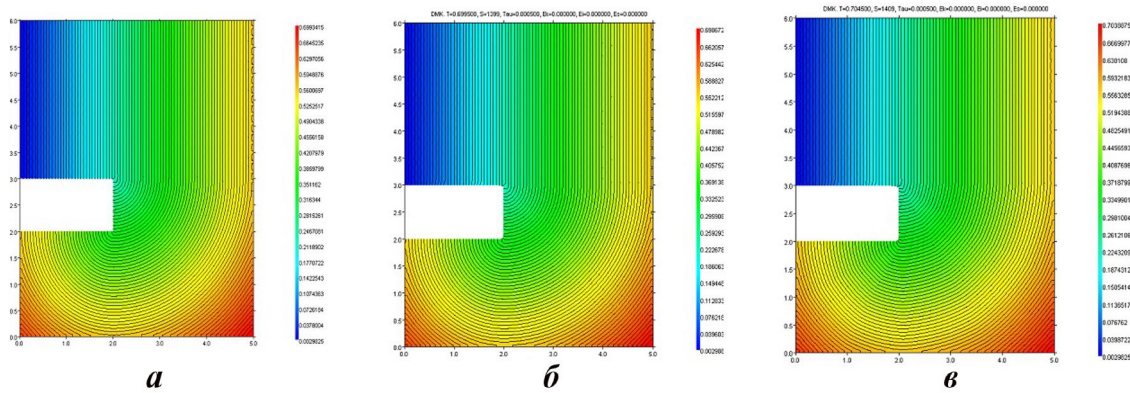


Рис. 8. Contour graphs of the detonation times calculated analytically (a) and by the exact (b) and step-by-step (c) control algorithms

**Propagation of a cylindrical detonation wave considering its passing over the inert layer.** Fig. 6 shows the initial geometry for the two-dimensional case with initiation set at a point located in the left bottom corner rather than along a segment. Fig. 9 shows the problem geometry in the three-dimensional case. In the HE layer, there is an opening filled with an inert substance. Initiation takes place along the lower left edge of the geometry (shown yellow).

The analytic solution to this problem is given in the form

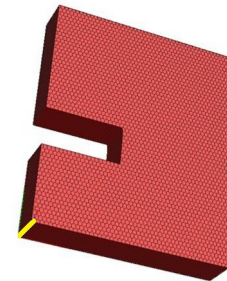


Fig. 9. Benchmark geometry for the three-dimensional case



$$T(x_0, y_0, z_0) = \begin{cases} \frac{\sqrt{x_0^2 + y_0^2}}{D} & \text{for } x_0 \in [0, 2], y_0 \in [0, 2], z_0 \in [0, 1]; \\ \frac{\sqrt{x_0^2 + y_0^2}}{D} & \text{for } x_0 \in [2, 5], y_0 \leq x_0, z_0 \in [0, 1]; \\ \frac{2\sqrt{2} + \sqrt{(x_0 - 2)^2 + (y_0 - 2)^2}}{D} & \text{for } x_0 \in [2, 5], y_0 > x_0, z_0 \in [0, 1]; \\ \frac{2\sqrt{2} + 1 + \sqrt{(x_0 - 2)^2 + (y_0 - 3)^2}}{D} & \text{for } x_0 \in [0, 2], y_0 \in [3, 6], z_0 \in [0, 1]. \end{cases}$$

Fig. 10 shows detonation front positions at the characteristic times calculated by the exact and step-by-step detonation front velocity control algorithms. Fig. 11 shows contour graphs of the detonation times calculated by the control algorithms and analytically.

Fig. 12 shows detonation time fields calculated by the step-by-step control algorithms and analytically.

The figures indicate that the results are in close agreement: The relative error of the detonation times calculated by the control algorithms with detonation time correction does not exceed 1% for all grid cells compared with the analytic solution. The relative error grows smaller with decreasing grid spacing.

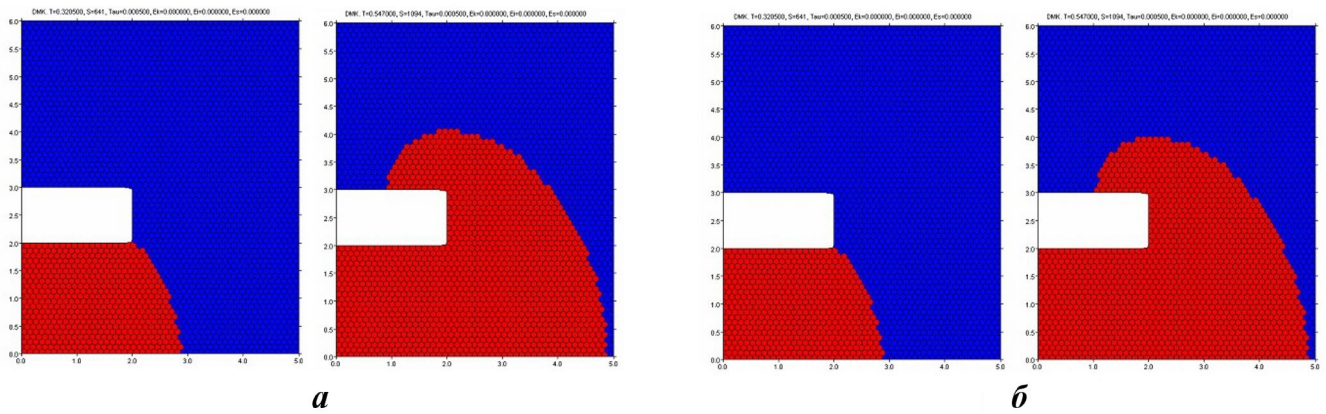


Рис. 10. Detonation front positions at the characteristic times calculated by the exact (a) and step-by-step (b) control algorithms: left —  $t = 0,3205$ ; right —  $t = 0,5468$

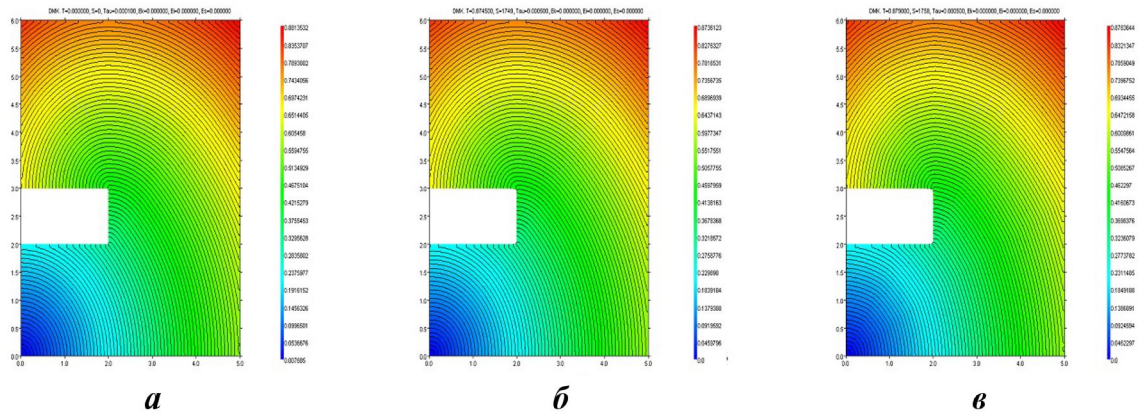


Рис. 11. Contour graphs of the detonation times calculated analytically (a) and by the exact (b) and step-by-step (c) control algorithms

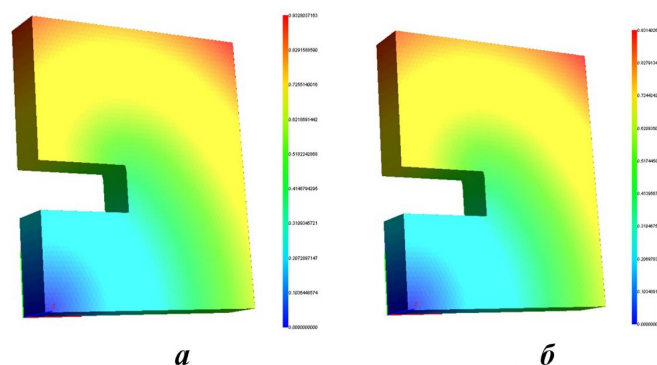


Рис. 12. Detonation time field calculated by the step-by-step (a) and analytically (b)

### Conclusion

In the TIM-2D and TIM codes, the process of steady detonation is calculated using detonation front velocity control algorithms. The control algorithms are divided by type into exact and step-by-step control algorithms, which can be used in different HE regions for high-precision applied simulations.

The advantage of the step-by-step detonation front velocity algorithm with detonation times corrected by considering the direction of motion of the detonation front is its generality and independence of the complexity of the initial geometry. The independence of the step-by-step control algorithm of the detonation front velocity enables calculations of detonation times for various multi-layer HE materials.

### Список литературы

1. Sokolov S. S., Voropinov A. A., Novikov I. G., Panov A. I., Sobolev I. V., Pushkaryev A. A. TIM-2D code for continuum mechanics simulations on arbitrary-connectivity irregular polyhedral grids // Voprosy Atomnoi Nauki i Tekhniki. Ser. Mathematical Modeling of Physical Processes. 2006. No. 4. P. 29–44.
2. Sokolov S. S., Panov A. I., Voropinov A. A., Novikov I. G., Sobolev I. V., Yalozo A. V. TIM code for three-dimensional continuum mechanics simulations on unstructured polyhedral Lagrangian grids // Voprosy Atomnoi Nauki i Tekhniki. Ser. Mathematical Modeling of Physical Processes. 2005. No. 3. P. 37–52.
3. Stanyukovich K. P. Detonation Physics. Moscow: Nauka, 1975.
4. Pushkaryev A. A., Sokolov S. S., Motlokhov V. N. Detonation velocity control algorithms in the TIM-2D code // Proc. VI Sc.-Tech. Conf. "Youth in Science". Sarov, October 30 –November 1 2007 Sarov: RFNC-VNIIEF, 2008. P. 165–171.
5. Bakhrakh S. M., Voronina E. B., Ustinova E. V., Shaverdov S. A. Laminar burning front propagation algorithm based on the Huygens principle // Voprosy Atomnoi Nauki i Tekhniki. Ser. Mathematical Modeling of Physical Processes. 1998. No. 1. P. 59–65.
6. Sofronov I. D., Vinokurov O. A., Zmushko V. V., Pletenev F. A., Izmaylova T. B., Potapkina L. F., Rybachenko P. V., Ryabikina N. A., Sokolova N. V., Trofimova L. Ya. MIMOZA-99 software system // Ibid. 1999. No. 4. P. 37–41.
7. Gavrilova E. S., Yanilkin Yu. V., Gorbenko A. D. Detonation velocity control program in the three-dimensional software system TREK++ // "Youth in Science". Proc. Second Sci.-Tech. Conf., Sarov, November 12–14, 2003 Sarov: RFNC-VNIIEF, 2004. P. 89–94.

Статья поступила в редакцию 06.11.20.